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Spectrum of Strange Mesons

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An energy eigenstates equation for mesons is derived and the energy levels of strange mesons are calculated and compared with those observed. For equal quark masses ($m_u = m_d$) the mass formula reduces to the mass formula describing nonflavored mesons with $I = 1$.

In a previous paper (Burcev, 1987), following the approach proposed by Kang and Schnitzer (1975), the energy levels of nonflavored mesons with $I = 1$ have been calculated using the Klein–Gordon equation with a box potential. In the model used, quarks behave like tachyons. In this paper I calculate the energy levels of strange mesons in a similar way.

Assuming that quarks behave like tachyons, the energy E in the center of mass of a system of a quark and an antiquark interacting by means of a potential V that behaves as the fourth component of a four-vector is ($c = \hbar = 1$)

$$E = (p^2 - m_1^2)^{1/2} + (p^2 - m_2^2)^{1/2} + V$$

where $p^2 = \mathbf{p}_1^2 = \mathbf{p}_2^2$ and m_1, m_2 are the masses of the quark and antiquark. Removing roots, we have

$$4(E - V)^2 p^2 - (E - V)^4 - 2(E - V)^2(m_1^2 + m_2^2) - (m_1^2 - m_2^2)^2 = 0$$

Making the usual quantum identifications $p^2 \to -\nabla^2$ and $E \to i\partial/\partial t$ and putting $\psi(\mathbf{r}, t) = \phi(\mathbf{r}) \exp(-iEt)$, we obtain

$$\left\{\nabla^2 + \frac{[(E - V)^2 + m_1^2 + m_2^2]^2 - 4m_1^2 m_2^2}{4(E - V)^2}\right\} \phi(\mathbf{r}) = 0$$

Assuming the confining potential in the form of a box potential,

$$V(t) = 0, \qquad 0 \leq r \leq R$$
$$V(r) = \infty, \qquad R \leq r$$

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we arrive at the equation for the energy eigenstates

$$(\nabla^2+p^2)\phi(\mathbf{r})=0, \qquad p^2=\frac{(E^2+m_1^2+m_2^2)^2-4m_1^2m_2^2}{4E^2} \tag{1}$$

Putting $\phi(\mathbf{r})=f(r)Y_m^l(\vartheta,\varphi)$ and solving equation (1) under the condition $f(r=R)=0$, we obtain

$$p_{lm}^2=(\alpha_{lm}/R)^2 \tag{2}$$

where α_{lm} are the roots of Bessel functions $J_{l+1/2}$ (Burcev, 1987). According to (1) and (2), we have

$$M_m^l=\{\alpha_{lm}^2K^2-M^2+[(\alpha_{lm}^2K^2-M^2)^2-(M^2-2m_1^2)^2]^{1/2}\}^{1/2} \tag{3}$$

with $M_n^l=E_n^l$, $K^2=2/R^2$, and $M^2=m_1^2+m_2^2$. For $m_1=m_2=m$, (3) reduces to

$$M_m^l=(\alpha_{lm}^2K^2-M^2)^{1/2} \tag{4}$$

with $K^2=4/R^2$ and $M^2=4m^2$. It was shown that the mass formula (4) together with

$$l<n, \qquad m=m_u=m_d=125\text{ MeV} \tag{5}$$

describes well the family of nonflavored mesons with $I=1$ (Burcev, 1987).

Now we will show that the mass formula (3) together with (5) describes the family of strange mesons. Putting $m_1=m$ and $m_2=m_s$, we can write (3) for a strange meson as

$$\begin{aligned} M_n^l&=\{\alpha_{lm}^2K^2-M^2+[(\alpha_{lm}^2K^2-M^2)^2-(M^2-2m^2)^2]^{1/2}\}^{1/2}\\ M^2&=m^2+m_s^2 \end{aligned} \tag{6}$$

We can fix the two free parameters M and K by choosing as input the masses of two different meson states. For the states $J^P=0^-$, 0^+, 1^+, and 2^- the value of l is defined uniquely. Inspecting the family of strange mesons, we find only three established pure states with l defined uniquely, namely $K(495.7)$, $l=0$; $K_0^*(1350)$, $l=1$; and $K_2(\sim 1700)$, $l=2$ (Particle Data Group, 1986). Therefore we take as input

$$\begin{aligned} M_2^0&=M(K)=495.7\text{ MeV}\\ M_4^1&=M(K_0^*)=1350\text{ MeV} \end{aligned} \tag{7}$$

[The assumption $M_1^0=M(K)$ does not agree with the meson states observed.] Then, according to (5) and (6), $K^2=4958\text{ MeV}^2$, $M^2=69{,}733\text{ MeV}^2$.

In Table I the energy levels M_n^l of strange mesons calculated according to (5)-(7) are compared with those observed (Particle Data Group, 1986).

Table I. Calculated and Observed Energy Levels M_n^l of Strange Mesons[a]

	M_n^l (MeV)					
l	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$J^{P\,2s+1}l_J$
0	496	860	1194	1519	1839	
	$K(\underline{496})$					$0^-\ {}^1S_0$
		$K^*(892)$				$1^-\ {}^3S_1$
1	670	1019	1350	1674	1994	
			$K_0^*(\underline{1350})$			$0^+\ {}^3P_0$
			$\underline{K_{1B}}$			$1^+\ {}^1P_1$
			$\underline{K_{1A}}$			$1^+\ {}^3P_1$
			$K_2^*(1426)$			$2^+\ {}^3P_2$
2		1168	1499	1823	2144	
						$1^-\ {}^3D_1$
						$2^-\ {}^1D_2$
				$K_2(\underline{1770})$		$2^-\ {}^3D_2$
				$K_3^*(1780)$		$3^-\ {}^3D_3$
3			1643	1969	2290	
						$2^+\ {}^3F_2$
						$3^+\ {}^1F_3$
						$3^+\ {}^3F_3$
				$K_4^*(2060)$		$4+\ {}^3F_4$

[a]States with l defined uniquely are underlined.

The M_1^0 calculated is not real. Therefore the state $(ln)=(01)$ is omitted from the table. States with l defined uniquely are underlined. The energy levels calculated agree with pure states observed within an accuracy of $\sim 5\%$. Let us note that for the mixtures $K_1(1280)$ and $K_1(1400)$ the mean mass 1340 MeV is close to $M_4^1 = 1350$ MeV.

For the mass of the s quark we have

$$m_s = (M^2 - m^2)^{1/2} = 233 \text{ MeV}$$

and the radius of the box $R = 2^{1/2}/K = 4.0\times 10^{-13}$ cm. In the models in question the radii of the boxes for strange mesons and for nonflavored mesons with $I=1$ are practically the same. For the difference of quark masses $\Delta = m_s - m$ we have $\Delta = 108$ MeV. This value is close to $\Delta \sim 120$ MeV found by Kokkedee (1969).

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